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## **PAPUA NEW GUINEA PRIMARY MATHEMATICS TEXTBOOK AND REFLECTION: A COMPARISON WITH JAPAN**

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### **Abstract**

This study examined how primary mathematics textbooks can foster reflective learning, filling a research gap. A literature search using the keywords “mathematics textbook” and “reflection” yielded 882 results, yet none directly linked the two. Consequently, this study focused on articles exploring terms related to reflection, specifically in the learners’ context. O’Keeffe’s (2013) textbook analysis framework was integrated with Hino’s (2013) model, yielding a new analytical tool with four elements (Content, Structure, Expectation, Language) and three strategies (Questioning, Polish-up, Merit of Mathematical Ideas). The analysis revealed that (1) expectations are often implied in Key Questions but rarely explicit; (2) questioning plays a significant role in prompting reflection for both teachers and students; and (3) conceptual differences in understanding fractions between textbooks from Papua New Guinea and Japan highlight the importance of textbooks as tools for promoting reflective learning. This study also compared the presentation of fractions in textbooks from both countries.

**Keywords:** Papua New Guinea, Mathematics Textbook, Reflection, Reflective Learner, Fractions

### **INTRODUCTION**

Papua New Guinea (PNG) is improving the quality of mathematics education, recognizing it as a major and fundamental subject in a highly technological society. PNG has received technical assistance from Japan through the Japan International Cooperation Agency (JICA) to enhance mathematics education. This collaboration focuses on developing and improving mathematics textbooks and acknowledges their key role in teaching. The initiative includes capacity-building programs for teachers, ensuring they are well equipped to utilize the improved textbooks and adopt effective teaching methodologies. The partnership extends to educational policy and curriculum development, aiming to strengthen PNG’s overall mathematics education framework (DOE and JICA, 2019).

This study examines the effectiveness of textbook quality and its analysis of reflection in enhancing teaching practices, supporting teachers’ professional development, and improving students’ learning outcomes, to observe the connection to improving understanding and

instruction. Reflection is essential in addressing the inherent and culturally contingent difficulties associated with fractions and decimal arithmetic. By reflecting on students' learning processes, educators can identify common misunderstandings, such as fraction and decimal notation or the opaqueness of arithmetic procedures. Similarly, reflection on culturally contingent factors, like teaching methods or prior knowledge deficits, allows for tailoring of interventions to address learners' specific challenges (Lortie-Forgues et al., 2015). This iterative reflection process ensures that instruction evolves to accommodate diverse student needs better, ultimately reducing barriers to mastering fractions and decimals. This study selects the fraction topic because it is a central concept in primary school mathematics, but has two key difficulties associated with fractions and decimals: (1) the inherent challenges of fraction and decimal arithmetic, such as the opaqueness of their procedures and the complex relationships between arithmetic operations, and (2) culturally contingent obstacles that can be mitigated through improved instruction and learners' prior knowledge (Lortie-Forgues et al., 2015). Reflection is critical, allowing educators and researchers to assess and address these difficulties effectively. By reflecting on teaching approaches and learners' conceptual gaps, strategies can be devised to foster better comprehension and reduce the complexity of fractions and decimals.

Inherent difficulties arise as fractions combine two numbers, such as a denominator and numerator, to represent one number, while they have various meanings, such as operation, quantity, and quotient. Apule et al. (2016) identified misconceptions regarding fractions among students and pre-service teachers in teachers' colleges in PNG. Early proficiency in fractions uniquely predicts success in advanced mathematics. Lortie-Forgues et al. (2015) highlighted the importance of the knowledge of rational numbers for progressing to higher-grade mathematics and later applications in occupational settings. The mitigation of culturally contingent obstacles is significant because it emphasizes the possibility of improving instruction by considering learners' prior knowledge. Teachers must be aware and reflective of learners' difficulties and prior knowledge.

Therefore, this study selected fractions as the topic for textbook analysis, aiming to identify how reflection can be attained in the textbook. The mathematical concepts focused on the addition and subtraction of fractions with common, equivalent, or different denominators—areas in which primary school students and student-teachers demonstrated significant challenges during testing. This difficulty is widespread and persistent, with notably low student performance on fraction problems, especially with different denominators (Lortie-Forgues et al., 2015, p. 204). Poor understanding of fractions severely undermines students' ability to grasp higher mathematical concepts, such as ratios, proportions, and algebra (Lortie-Forgues et al., 2015, p. 203). Reflection is pivotal in addressing these deficiencies by allowing learners to identify and overcome conceptual misunderstandings, such as errors stemming from over-reliance on whole-number reasoning, potentially yielding improved strategies and long-term comprehension (Lortie-Forgues et al., 2015). Without addressing these gaps in understanding, advancing to more complex mathematical concepts becomes exceedingly

difficult. Given the demonstrated difficulties faced by primary school students and student-teachers in addition and subtraction of fractions with common, equivalent, or different denominators, reflection presents a valuable pedagogical approach to improving conceptual understanding. Reflection fosters metacognitive awareness, enabling learners to critically examine their problem-solving strategies, identify misconceptions, and refine their reasoning processes. Research highlights the role of reflective practices in enhancing mathematical cognition by promoting deeper engagement with mathematical structures and fostering self-regulated learning (Lortie-Forgues et al., 2015). Integrating structured reflection into fraction instruction, such as reflective journaling, guided self-explanation, and peer discussions, can strengthen conceptual foundations, mitigate procedural errors, and improve retention of fraction concepts. Thus, embedding reflective practices in mathematics education holds significant potential for addressing foundational gaps and advancing learners' proficiency in fractions. Reflection in textbooks can promote the development of learners' deep understanding of fractions. We posed the following three research questions:

RQ1. What is textbook analysis and reflection in previous research projects and this study?

RQ2. What analysis framework do the mathematics textbooks use to promote learners' reflection?

RQ3. What conclusions can be drawn—regarding the encouragement of reflection—from analyzing a mathematics textbook in PNG versus one in Japan?

A literature review was conducted using the keywords “mathematics textbook” and “reflection.” A search of international scientific databases yielded 882 studies on either textbook analysis or reflection. However, no study has addressed textbook analysis as a means of promoting reflection. This highlights the need to develop a method for textbook analysis from this perspective, identifying Japanese textbooks as ideal tools for clarifying the characteristics of PNG textbooks. The first reason is that the Japanese lesson is characterized as structured problem-solving (Stigler et al., 1999), which assumes reflection during the lesson. Textbooks and lessons are closely related. Second, textbooks in Japan are not national textbooks but are screened by the ministry. This is similar to the situation in PNG, although the Grade 4 Mathematics textbook in PNG is a national textbook. Third, the Japanese government has supported the PNG government to improve the mathematics textbook. Indeed, staff and teachers are familiar with this kind of textbook.

This study employed a structured comparative analysis framework to examine how mathematics textbooks foster reflection among primary school learners. Integrating O’Keeffe’s (2013) four-element model of Content, Structure, Expectation, and Language, in addition to incorporating Hino’s (2013) reflective learning strategies, this study systematically evaluated textbook design, specifically its potential to encourage active learning. By comparing Japanese and PNG mathematics textbooks, this study highlighted key pedagogical differences and identified strategies for enhancing reflection within instructional materials. This rigorous approach ensures a comprehensive understanding of the role of textbooks in

promoting reflective learning and contributes to broader discussions on curriculum development and educational equity.

## **LITERATURE REVIEW**

This section explores the key theoretical foundations essential for understanding how mathematics textbooks foster reflective learning.

## **REFLECTION**

Reflection is critical in mathematics learning; simply completing tasks is insufficient. The role of communication and assessment in problem-centered learning, and its effect on reflection, are considered. (Wheatley, 1992). We must encourage students to reflect on their activity. For example, being asked to justify a method of solution often promotes reflection (Wheatley, 1992). Reflection is important for deepening and consolidating learning quality.

Second, the literature illustrates the teaching approach to reflection. Limjap (2013) stated that teachers' pedagogical content knowledge was applied through a constructivist lesson plan to promote reflective thinking among elementary pupils (p. 306). Voon (2013) suggested that reflective learning by encouraging habitual practice of reflection can be applied to any subject (p. 27). Other studies have highlighted factors influencing how reflection is cultivated in learners, such as Boaler (2016) on reflective journaling and Larrivee (2009) on how such practices encourage reflection in learners.

In this research, "reflection" implies educational chances for developing better understanding and high-level cognition by learners (Hino, 2013, p. 265), and promotion of reflection is regarded as a critical teaching strategy.

Both of the above studies explored ways to encourage reflective learning in math, but mainly focused on classroom interactions and descriptions rather than testing ideas across different settings (Hino, 2013; Koyama, 2013). They also failed to fully consider individual learning differences or the external factors potentially affecting reflection (Hino, 2013; Koyama, 2013).

## **RELATION BETWEEN TEXTBOOKS AND PROMOTION OF REFLECTION**

Textbooks serve as crucial bridges between intended and implemented curricula, functioning as the potentially implemented curriculum (Fan et al., 2013; O'Keefe, 2013). Their quality directly influences how students engage in reflective learning during lessons, shaping their ability to critically analyze mathematical concepts and problem-solving approaches.

Figure 1 was used by O'Keefe (2013) for textbook analysis. It describes the connection between intended and implemented curricula through a potentially implemented curriculum whereby contents and their arrangement of textbooks are closely related to lesson topics and

their sequences.

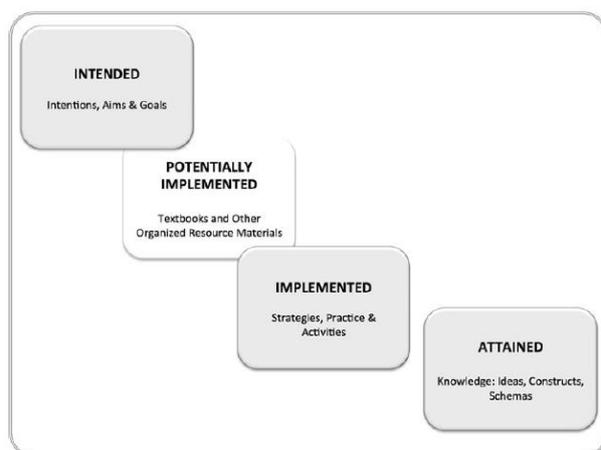


Figure. 1. Tripartite Model developed by Valverde et al. 2002 used by O’Keeffe (2013)

Source: O’Keeffe (2013, p. 3)

Two additional research projects, from Koyama (2013) and Hino (2013), are adopted to concretize students’ reflection in the lesson. Both discussed how to promote reflection in the lesson. Koyama (2013) suggested four methods for eliciting learners’ reflections: (1) paying attention to the correctness of learners’ answers as well as how students arrive at the answer; (2) providing guidance to learners on image-making and how it can be used for mathematical understanding; (3) questioning techniques, which help students develop reasoning and higher levels of mathematical understanding; and (4) using prior learning to recall knowledge and employ it in difficult learning situations. Koyama (2013) did not directly mention the use of textbooks. However, because of the position of the textbook between intended and implemented curricula, his points are expected to appear in the Japanese textbook’s sequencing.

Hino (2013, p. 254) suggested three ways of using mathematics textbooks to promote reflection. First, noticing and questioning, that is, creating learning situation and an activity, are encouraged. Second, learning is polished through group work or collaborative efforts, and the third merit of mathematical ideas is the provision of reflection among learners.

## PERSPECTIVES OF TEXTBOOK ANALYSIS

Textbook analysis is critical for identifying how instructional methods are embedded in curricular materials and realized in classroom practice. O’Keeffe (2013) synthesized prior research and established a comprehensive framework for evaluating textbooks, drawing from Halliday (1973), Rivers (1990), Morgan (2004), and the TIMSS (Valverde et al., 2002). Through this synthesis, O’Keeffe (2013) identified four key elements of textbook analysis: Content, Structure, Expectation, and Language. Each is vital in determining how textbooks influence learning outcomes and curriculum implementation.

Recent studies, including Johansson (2003), Peppin and Haggarty (2001), and Afolabi (2014), align closely with the TIMSS study (Valverde et al., 2002), reinforcing the latter's relevance in contemporary textbook research. While Rivers (1990) and Mikk (2000) incorporated some of these elements, Morgan (2004) focused exclusively on language analysis, highlighting its distinct contribution to textbook evaluation. Consequently, both the TIMSS study and O'Keeffe's (2013) framework are regarded as the most comprehensive approaches to textbook analysis (Table 1).

Table 1. Summary of Theoretical Frameworks developed and revised for textbook analysis.

<b>Studies</b>	<b>Function</b>	<b>Considered elements</b>
Valverde et al. (2002)	Provides overall structure for textbook analysis	Significant for three key elements: 1. Content Analysis 2. Structural Analysis 3. Expectation Analysis
O'Keeffe (2013)	Allows for a better understanding of the difficulty of the mathematical language as encountered by students using textbooks	Significant for four key elements: 1. Structural Analysis 2. Content Analysis 3. Expectation Analysis 4. Language Analysis

Source: Developed by the author based on O'Keeffe (2013, p.10)

O'Keeffe (2013) revised the TIMSS textbook analysis framework (Valverde et al., 2002) and defined four key elements: content analysis, structure analysis, expectation analysis, and language analysis.

In this study, textbook analysis builds on previous studies, utilizing such frameworks as the TIMSS study to examine how textbooks serve as instruments for curriculum implementation. While prior analyses have primarily focused on the structural alignment between textbooks and curricular objectives, this study extended the scope by exploring how textbooks actively support educational goals through O'Keeffe's (2013) four elements.

For RQ1, we drew on previous studies to highlight the role of reflection in learning, particularly in mathematics. Reflection enables students to articulate and justify their answers, fostering deeper understanding. Such strategies as noticing, questioning, polishing up, and recognizing the merit of mathematical ideas are vital to promote reflective thinking.

This research explored how textbooks can promote reflection in learners by analyzing textbooks. Textbooks connect the intended and the implemented curriculum, serving as a tool for content delivery and a medium for reflection and deeper understanding (DOE and JICA 2019). This study aimed to understand how textbook analysis and reflective learning strategies can be used to encourage critical thinking and conceptual engagement.

## METHOD

This study is grounded in constructivist epistemology, which views knowledge as something students actively build through their learning experiences rather than passively receiving it (Ernest, 1994; Olssen, 1995). Reflection is crucial in this process, allowing students to engage more deeply with mathematical concepts, think critically, and refine their reasoning skills.

Mathematics textbooks constitute a key educational tool connecting curriculum objectives to actual classroom learning (Valverde et al., 2002). By providing opportunities for reflection, textbooks can help students make meaningful connections between mathematical ideas and improve their understanding and problem-solving skills (Hino, 2013). A well-designed textbook encourages questioning, discussion, and structured learning, making reflection an essential component of mathematics education.

To examine how textbooks foster reflective learning, this study applied O’Keeffe’s (2013) textbook analysis framework with four elements: Content, Structure, Expectation, and Language. This framework was combined with Hino’s (2013) reflective learning strategies, which emphasize critical thinking and deeper engagement with mathematical concepts. By integrating these approaches, this study systematically analyzed how textbooks from PNG and Japan encourage reflection among students.

Based on the answers to RQ1, this study employed a qualitative content analysis approach to examine the Structure, Content, Expectation, and Language of mathematics textbooks from PNG and Japan. The analysis was guided by O’Keeffe’s (2013) framework for textbook evaluation, which incorporates insights from Halliday (1973), Morgan (2004), Rivers (1990), and the TIMSS study (Valverde et al., 2002). By assessing how textbooks mediate learning, the current study identified patterns in instructional design that support reflection and conceptual understanding.

Accordingly, a comparative textbook analysis framework was developed to ensure alignment with these elements of textbook analysis and apply them to specific components of the textbooks. The primary objective was to promote reflection on learning by identifying how textbook design influences students’ engagement with mathematical concepts.

This study incorporated three strategies for nurturing reflective learning (Hino, 2013), based on the assumption that the structure and content of textbooks serve as a mechanism for fostering reflection. Combining O’Keeffe’s textbook analysis framework with Hino’s reflective learning strategies yields a structured method for textbook evaluation (Table 2). The table illustrates how Content, Structure, Expectation, and Language are integrated into strategies to promote reflection, providing a comprehensive framework for enhancing learners’ engagement with mathematical concepts.

Table 2. Textbook analysis for Nurturing Reflection

<b>Elements of Analysis</b>	<b>Perspective Analysis</b>	<b>of Target analysis</b>	<b>of</b>	<b>Strategy Noticing, Questioning, Key Question</b>	<b>Polish-up, Deliberation on Key Question</b>	<b>Merit of Mathematical Ideas Discussion</b>
<b>Content analysis</b>	How (content) are sequenced for reflection. Sequence of questions.	topics are for	Content	1-1	1-2	1-3
<b>Structural analysis</b>	How structure promotes reflection and type of question. Diagram regarding questions.	structure of	Diagrammatic	2-1	2-2	2-3
<b>Expectation analysis</b>	How aims, goals, objectives are stated in relation to reflection. Interpretation of questions.	aims, goals, [objective] Implied		3-1	3-2	3-3
<b>Language analysis</b>	How mathematical symbols, notational signs and words signifiers are used for reflection. Symbols of questions.	mathematical symbols and words signifiers are used for reflection. Symbols of	Symbol	4-1	4-2	4-3

Source: Developed by the author for analyzing textbooks and reflection.

We conducted a textbook analysis comparing PNG and Japanese mathematics textbooks. We selected them based on their relevance to the national curricula and their role in shaping instructional practices. The textbooks analyzed are:

- PNG textbook (2019)
- Japanese textbook (2012)

The analysis focused on Mathematics Grade 4 and the topic of fractions, examining the key pedagogical elements that contribute to effective learning and reflection. These elements include:

- Content sequencing, which assesses how topics are structured to facilitate comprehension and foster reflective thinking.
- Diagram placement, analyzing how visual representations emphasize Key Questions that guide student inquiry.
- Expectation alignment, evaluating how textbooks introduce and reinforce lesson objectives by connecting new content to prior knowledge.
- Language use, investigating how mathematical symbols and terminology support conceptual understanding and thinking skills.

The definitions and descriptions of each element, along with strategies for nurturing reflection, were systematically applied to both textbooks to ensure a structured and comparative evaluation.

(1) Content analysis was employed to systematically examine how mathematics textbook topics are sequenced and structured to facilitate learning and reflection. This approach emphasizes the organization of instructional components, such as questions, discussions, and summaries, which shape students' engagement with mathematical concepts. By analyzing these textual features, this study sought to identify patterns in how textbooks support thinking or reasoning, encourage reflective thinking, and align with pedagogical goals. This method provides a structured framework for evaluating the effectiveness of textbooks in fostering deeper comprehension and critical engagement with mathematical ideas.

A Key Question introduces major content in the lesson, Polish-up refines the question for understanding content, and Discussion deliberates on questions for summary.

### **1-1 [Noticing/ Questioning]**

Both textbooks have Key Questions:

Japanese: "How many m should we say Misaki's and Kenta's arm spans are?"

PNG: "What are the amounts of water in Molly's bottle and Steven's bottle in litres, respectively?"

Key Questions are explicitly posed to students. A Key Question requires individual learners to consider the current topic by reflecting on the previous lesson.

**1-2 [Polish-up]**

The characters and/or children in the explanation are used to ask questions, making it possible to deliberate on the Key Question and hold dialogue between two or more characters/children. This dialogue shows reflection for the Polish-up of content to deepen understanding. It prompts the next questions:

Japanese: "How can we express the length of Misaki's arm span in m?"

PNG "What are the amounts of water in Molly's bottle and Steven's bottle in litres, respectively?"

**1-3 [Merit of Mathematical Ideas]**

The discussion denotes whether the characters discuss and realize certain mathematical ideas:

Japanese: "Some fractions are larger than 1."

PNG There is a refined problem, which imply the existence of fractions larger than 1.

(2) Structure analysis in this study examined how the placement of diagrams within mathematics textbooks reinforces key instructional questions, specifically those framed as "what" or "how" inquiries. These visual elements serve as cognitive anchors, guiding students toward a deeper engagement with mathematical concepts. The notion of Polish-up was explored through deliberation questions embedded in animations, discussions, and activities, fostering reflective thinking and conceptual refinement. By analyzing these structural components, the current study aimed to determine how textbooks facilitate comprehension, encourage inquiry-based learning, and support the development of mathematical reasoning. This approach ensured that the organization of textbook content aligned with pedagogical objectives, ultimately enhancing students' ability to critically engage with mathematical ideas.

**2-1[Noticing/Questioning]**

Diagrams accompany the Key Question:

Japanese: As an introduction it shows a camera picture with a girl expanding her arms. This is followed by a diagram, just before the Key Question, focused on different unit lengths. These diagrams promote understanding of the Key Question.

PNG After the Key Question, there are the diagrams of two bottles and schematic representations corresponding to the bottles.

**2-2 [Polish-up]**

In relation to the Key Question, some diagrams are used to consolidate understanding of the Key Question and others to refine it.

Japanese: After the Key Question, the diagram shows Misaki's arm in terms of various unit lengths. It prompts the next "refined Key Question." Another diagram supports this question.

PNG After the Key Question, two children discuss the diagrams.

**2-3 [Merit of Mathematical Ideas]**

After the Polish-up of the discussion, we may find not only the answer but also the important point of mathematical ideas.

Japanese: After refining the Key Question, one diagram shows the point  $\left(\frac{5}{4}\right)$  m more clearly, and the last diagram comparatively shows  $\frac{3}{5}, \frac{5}{5}, \frac{7}{5}$  m.

PNG There is a refined question and diagrams showing 1l and  $\frac{1}{3}$  of water.

(3) Expectation analysis in this study examined how a Key Question functions as both an instructional trigger and thinking prompt, serving the dual purpose of articulating lesson objectives and activating students' prior knowledge. By engaging learners in recalling previously learned concepts, the Key Question fosters continuity in understanding and prepares them for deeper exploration of new material. The Polish-up component reinforces lesson objectives through structured follow-up questions that encourage deliberation on the Key Question, facilitating further discussion and interactive activities. This iterative process strengthens comprehension and ensures that students systematically refine their understanding, culminating in a well-structured summary that consolidates learning outcomes and aligns with the intended curriculum goals. By using this approach, textbooks can effectively scaffold student engagement, supporting reflective learning and sustained knowledge retention.

**3-1 [Noticing/ Questioning]**

The type of key question indicates what level of objective the topic asks for in the lesson. It may simply recall some facts, find an answer for the calculation, consider ways of finding solutions, and so on. However, since this is a prompt for learners to engage themselves with mathematical activities, the interpretation is necessary.

Japanese: The Key Question “How many m should we say Misaki’s and Kenta’s arm spans are?” with the diagram calls attention to arm spans and length. It is assumed that the children had already learned the concepts of length and unit length; however, length here cannot be explained in terms of unit length. Here, they were expected to recall unit lengths ( $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{1}{4}$  m) and think more extensively using unit length.

PNG The Key Question “What are the amounts of water in Molly’s bottle and Steven’s bottle in litres, respectively?” with the diagram calls attention to bottles of water and volume. The children had already learned the concept of the volume of water, but the volume here cannot be explained in terms of unit volume ( $\frac{1}{3}$ l). They were expected to recall the unit volume and think more about using it.

### **3-2 [Polish-up]**

The type of refined question indicates the level of detail of the lesson’s objective. Again, this is the next prompt and needs to be interpreted.

Japanese: The refined Key Question “How can we express the length of Misaki’s arm span in m?” with the diagrams calls for more specific attention to the unit length ( $\frac{1}{4}$ m). The objective here is to use the unit length ( $\frac{1}{4}$ m). Here, they may be expected to recall how to use the unit length (1m) as well, but this does not appear in the textbook. They must think more by connecting these facts with each other. This finding suggests higher levels of cognitive ability.

PNG The refined question or prompt “Let’s think about how to represent fractions larger than 1 and how can we calculate it” with the diagram calls more specific attention to the unit volume ( $\frac{1}{3}$ l). They may also be expected to recall how to use the unit volume (1 l), but this does not appear directly.

### **3-3 [Merit of Mathematical Ideas]**

After the discussion, the refined question and the characters’ dialog denote some mathematical ideas.

Japanese: The mathematical idea in the end is an extension of the fraction beyond 1. The extension of number concept is one of the important mathematical ideas in the number domain.

PNG The mathematical idea which also emerges in the middle of the lesson concerns a fraction beyond 1. The extension of the number concept is one of the important mathematical ideas in the number domain.

(4) Language analysis in this study examined how the Key Question incorporates mathematical symbols to facilitate conceptual understanding. These symbols serve as cognitive tools that help students interpret and engage with mathematical ideas more effectively. In the Polish-up phase, follow-up questions build upon the Key Question, encouraging deeper deliberation and reinforcing comprehension through structured discussions and activities. The integration of mathematical symbols throughout these stages ensures the continuity of learning and guides students from inquiry to application. The presence of symbols in summaries consolidates knowledge, reinforcing key concepts and promoting reflective thinking. By analyzing the role of language and symbolic representation, this study highlighted how textbooks support thinking skills and foster meaningful learning experiences.

#### **4-1 [Noticing/Questioning]**

Attention is paid to symbols, notational signs, etc., in the Key Question.

Japanese: Before the Key Question, there are notations, such as  $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{1}{4}$ , but not in the Key Question. The children have learned these notations and are expected to recall them.

PNG After the Key Question are notations, such as 1 l and  $\frac{1}{3}$  l. The children have learned these notations and are expected to recall them.

#### **4-2 [Polish-up]**

The refinement was undertaken while the Key Question was discussed. The discussion called specific attention to the point by using the mathematical notations and symbols.

Japanese: The refined Key Question contains the mathematical symbol ( $5\frac{1}{4}$ ). Thus, refinement is undertaken in terms of mathematics.

PNG The refined Key Question includes the phrase “fraction more than 1.”

#### **4-3 [Merit of Mathematical Ideas]**

The refined question and the characters’ discussion included some mathematical ideas but not notations.

Japanese: The merit of mathematical ideas in the end is an extension of the fraction beyond 1. It does not include mathematical notation.

PNG: The mathematical notation at midpoint in the lesson is used in place of “fraction

more than 1.”

Numbers of the textbook pages in Table 2 and Figures 2 and 3 are arranged as follows: 1-1, 1-2, and 1-3 refer to content; 2-1, 2-2, and 2-3 refer to diagrammatic; 3-1, 3-2, and 3-3 refer to objective; and 4-1, 4-2, and 4-3 refer to symbols. They are used to indicate the features of the four elements of textbook analysis combining the three strategies for nurturing reflection to facilitate the textbook analysis. At the end of each number, J refers to Japanese and P refers to PNG. As Table 2 shows, we analyzed each textbook per element and strategy. Thus, 3-2 P in Figure 3 indicates expectation analysis (“3”), Polish-up strategy (“2”), and PNG textbook (“P”).

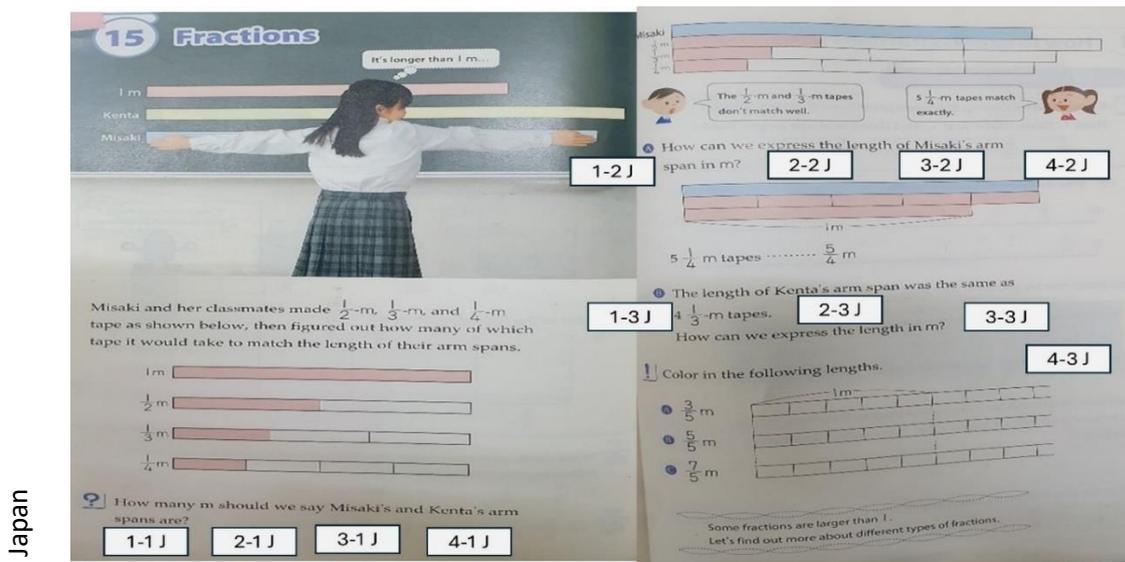


Figure 2. Japanese Textbook lesson topic: Fractions

Source: Keirinkan (2012, pp. 66–67)

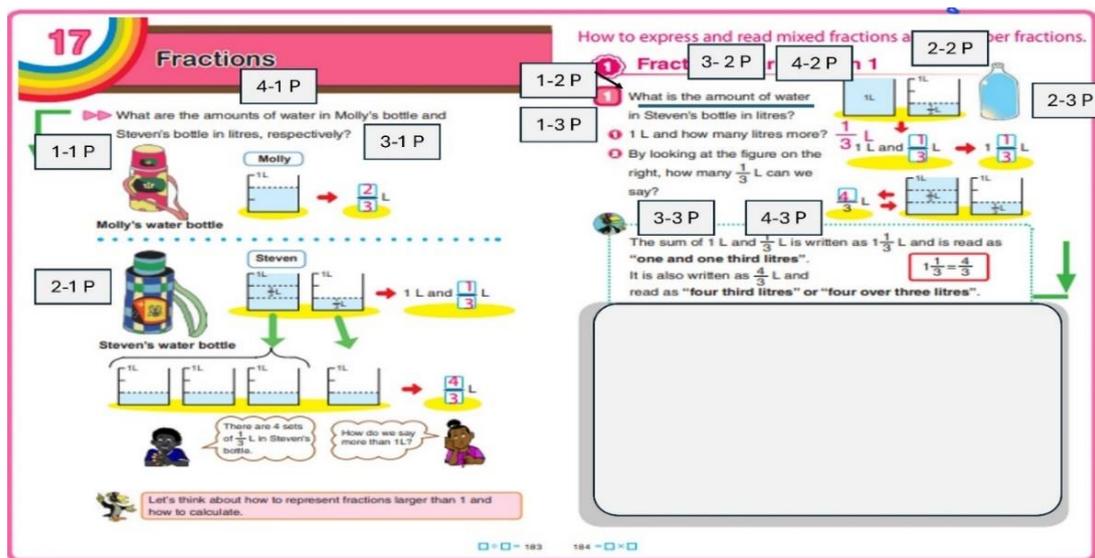


Figure 3. PNG Textbook lesson topic: Fractions

Source: Department of Education (2019, pp. 183–184)

## FINDINGS

Using the textbook analysis framework (Table 2), we combined the four elements of textbook analysis (O’Keeffe, 2013) with the three strategies for nurturing reflective learners (Hino, 2013), for the analysis.

Tables 3 and 4 summarize the results of the comparative analysis of Japanese and PNG textbooks. The analysis was conducted using four key elements (Content, Structure, Expectation, and Language) against three strategies for nurturing reflection (Key Question, Deliberation on the Key Question, and Discussion) to encourage reflection.

Table 3. Results of Japanese textbook

	Key Question	Deliberation on Key Question	Discussion
<b>Content analysis</b>	1-1 Key Question is asked after students demonstrate length of arm spans in meters (m).	1-2 A follow-up question is asked in deliberation of the key question to express length of arm spans in meters.	1-3 Discussion and activities for understanding immediately after the follow-up question.
<b>Structural analysis</b>	2-1 Before Key Question is asked, a picture of students demonstrates length of arm span.	2-2 As a follow-up question for deliberation of the Key Question referring to tape diagram to explain lengths in meters (m).	2-3 The discussion and activities utilize diagrams for understanding.
<b>Expectation analysis</b>	3-1 Objectives of the lesson are observed through asking of key question.	3-2 Maintenance of the lesson objective is determined through a follow-up question for deliberation of the Key Question to explain lengths in meters (m).	3-3 Through discussions and completion of activities with summarization, the lesson objective is achieved.
<b>Language analysis</b>	4-1 The Key Question contains symbols and terms.	4-2 The deliberation of the Key Question also contains symbols and terms.	4-3 The discussion and activities contain symbols and terms.

Source: Developed by the author for analysis purposes

Table 4. Results of PNG textbook

	<b>Key Question</b>	<b>Deliberation on Key Question</b>	<b>Discussion</b>
<b>Content analysis</b>	1-1 The Key Question to introduce the lesson comparing amounts of water in the bottle in liters.	1-2 A follow-up question is asked in deliberation of the Key Question to compare amounts of water in the bottle in liters.	1-3 Discussion and activities for understanding immediately after the follow-up question.
<b>Structural analysis</b>	2-1 The Key Question is asked and below it a diagram display amounts of water.	2-2 A follow-up question for deliberation of the Key Question is asked with diagrams to explain amounts of water in liters (I).	2-3 The discussion and activities utilize diagrams for understanding.
<b>Expectation analysis</b>	3-1 Objectives of the lesson is observed through asking of the Key Question.	3-2 Maintenance of the lesson objective is determined through a follow-up question for deliberation of the Key Question referring to amounts of water in liters (L).	3-3 Through discussions and completion of activities with summarization, the lesson objective is achieved.
<b>Language analysis</b>	4-1 The Key Question contains symbols and term.	4-2 Deliberation of the Key Question also contains symbols and terms.	4-3 The discussion and activities contain symbols and terms.

Source: Developed by the author for analyzing reflection

To answer RQ3, we conducted a comparative analysis of Japanese and PNG textbooks on the topic of fractions, highlighting critical aspects of their teaching approaches. The following subsections are summarized in Tables 3 and 4.

## CONTENT ANALYSIS

First, we noted the following aspects of the content analysis for comparison. In the Japanese textbook, the Key Question is posed after students physically demonstrate arm span lengths in the middle of the lesson, followed by the deliberation on the Key Question to express arm span in meters (m). Immediately after the deliberation question, discussions and activities are conducted to reinforce understanding by utilizing tape diagrams. By contrast, the PNG textbook introduces a Key Question immediately to engage students by comparing the amount of water. This discussion is then used to solidify understanding. Therefore, the timing and placement of the Key Question differs between textbooks, with the Japanese textbook

embedding it in the lesson and the PNG textbook introducing it at the start.

### **STRUCTURAL ANALYSIS**

Second, we performed a structural analysis for comparison. In the Japanese textbook, before the key question is asked, a photograph of a student demonstrating arm-span lengths is used to further students' understanding of familiar subjects or objects. Deliberation of the Key Question is developed using a tape diagram to illustrate this concept. The said approach uses tape diagrams to enhance understanding through discussions and activities. Conversely, the PNG textbook presents a Key Question accompanied by diagrams of objects not commonly used by students, showing the amount of water. Consequently, the students take time to understand the concept. The deliberation of the Key Question uses diagrams to explain the water measurements. Immediately after the follow-up deliberation question, the discussion uses these diagrams as guides to help students understand the concept.

### **EXPECTATION ANALYSIS**

Both textbooks embed lesson objectives in the Key Question. In the Japanese textbook, the Key Question encapsulates the lesson objectives, and the deliberation of the Key Question reinforces these objectives, ensuring they are achieved through discussions and activities. Similarly, the PNG textbook focuses on lesson objectives through its Key Question and follow-up deliberations. However, the level of thinking or reasoning expected from students may differ slightly, with the Japanese textbook prompting higher-order thinking through its questioning sequence and the use of familiar real-life examples, thus providing opportunities to promote reflection.

### **LANGUAGE ANALYSIS**

Both textbooks use mathematical symbols and terms in the Key Question, Deliberation of the Key Question, and Discussion. This consistent use of language helps familiarize students with mathematical concepts and terminology. In both cases, symbols and terms are effectively integrated into the learning process, ensuring students are introduced to mathematical language early and repeatedly, reinforcing understanding and promoting reflection.

### **KEY INSIGHTS**

These observations provided key insights into the textbook analysis, and the nurturing of reflection strategies focused on each textbook to identify and encourage reflection by highlighting similarities and differences in their approaches to teaching core concepts of fractions.

1. The Japanese textbook adopts a practical approach, beginning with a real-world demonstration likely to be relatable to students. The PNG textbook, conversely, starts with comparative questions, offering a more abstract introduction to the lesson. The placement of the Key Question within the lesson structure is critical, with the Japanese textbook positioning it after a demonstration while the PNG textbook uses it at the outset.
2. Both textbooks extensively use diagrams and symbols, suggesting visual aids are essential for helping students comprehend mathematical concepts. Diagrams are significant in reinforcing understanding, with Japanese textbooks focusing more on tape diagrams and PNG textbooks using schematic representations of objects such as bottles.
3. Deliberation questions are essential in both textbooks to ensure the key question is thoroughly explored and understood. These deliberation questions are carefully designed to embed the lesson objectives and encourage students to engage more deeply with the material.
4. The consistent use of mathematical language and symbols in both textbooks is critical in promoting mathematical literacy. By introducing these elements early in the lesson, students are gradually introduced to the formal language of mathematics, which is reinforced through activities and discussions.
5. The Key Question is the core component of the textbook analysis. It connects content, structure, expectations, and language, serving as a guide for both teachers and students. The Key Question is instrumental in motivating a deeper engagement with mathematical concepts and promoting reflection.

The diagram related well to the Key Question by providing additional material for questioning. This objective was adequately addressed by the key question. The symbol was carried well in the Key Question to express length in meters and volume in liters (l). However, we observed that noticing was not featured; rather, questioning was frequently used to extract information to observe the promotion of reflection.

Polish-up observations were made vertically to deliberate on the Key Question. These observations sat well with the content, as they sought a deeper understanding of the topic. The diagram related well to the deliberation of the Key Question, and gave a visual perspective on sense to facilitate understanding of the topic. The objective was observed with the refined questions, and symbols were clearly used in the refined questions. The polishing aligned well with this observation. The merit of mathematical ideas observations were made vertically to gain a perspective on how discussions and follow-up activities were observed. The content discussion was clear because, prior to discussions, there was a refined question. The diagram supported learners' understanding. The objective was observed and discussed, and symbols were utilized in the discussion and activities to deepen students' understanding

of fractions beyond 1.

## **DISCUSSION**

This study systematically compared how mathematics textbooks in PNG and Japan foster reflective learning in fractional instruction. The analysis highlighted how textbook design influences students' engagement and cognitive development.

## **THE ROLE OF REFLECTION IN MATHEMATICS LEARNING**

Reflection is fundamental in enhancing students' conceptual understanding and problem-solving skills in mathematics. Wheatley (1992) and Limjap (2013) emphasized the importance of encouraging students to justify their problem-solving methods and foster deeper engagement with mathematical concepts. The Japanese textbook integrates reflective learning through structured problem-solving and embedded questioning techniques that guide learners in constructing mathematical meanings. By contrast, the PNG textbook primarily focuses on factual recall, with less emphasis on scaffolding reflective thinking. As Lortie-Forgues et al. (2015) noted, reflection mitigates culturally contingent difficulties in fractional arithmetic, reinforcing the need for textbooks to explicitly incorporate reflective learning strategies.

## **FINDINGS FROM THE COMPARATIVE TEXTBOOK ANALYSIS**

A comparative analysis revealed key differences in how each textbook promotes reflection, reinforcing the impact of pedagogical structures on thinking skills.

- **Placement of the Key Question:** Japanese textbooks present Key Questions within structured learning experiences, encouraging reflection through problem-solving (Stigler et al., 1999). By contrast, the PNG textbook introduces Key Questions at the beginning, limiting opportunities for learners to recall prior knowledge and engage in meaningful reflections.
- **Use of Diagrams and Visual Aids:** Both textbooks employ diagrams as instructional tools, yet their effectiveness in promoting reflection differs. Japanese textbooks integrate diagrams into structured problem-solving discussions, reinforcing conceptual understanding through questioning techniques (Koyama, 2013). Conversely, the PNG textbook contains diagrams, but their placement lacks the deliberate scaffolding that would deepen reflective learning and comprehension.
- **Thinking Skills and Expectation Alignment:** The Japanese textbook challenges students to extend their conceptual understanding by analyzing fractions beyond 1, incorporating prior learning as a foundation for reflection (Apule et al., 2016). The PNG textbook covers fractional concepts but does not consistently encourage

deeper thinking skills through structured reflection prompts, potentially limiting students' ability to connect mathematical ideas across lessons.

- **Mathematical Language and Symbolism:** The structured use of mathematical language supports reflective learning by helping students articulate their reasoning skills. As Morgan (2004) and O'Keeffe (2013) emphasized, textbooks function as vehicles for mathematical discourse, shaping students' comprehension through structured language. This study found that the Japanese textbook systematically integrates symbolic notation, reinforcing conceptual learning through iterative questioning, whereas the PNG textbook employs mathematical terms but lacks structured follow-up questioning to ensure reflective comprehension.

## **IMPLICATIONS FOR TEXTBOOK DEVELOPMENT**

The findings underscore the significant influence of textbook design on students' abilities to engage in reflective learning. The structured sequence of inquiry and scaffolded discussions in Japanese textbooks aligns with pedagogical models that emphasize problem-solving (Hino, 2013). By contrast, the PNG textbook would benefit from greater integration of reflective prompts and structured questioning, ensuring that students actively engage in reasoning processes rather than relying on factual recall.

This study reinforces the importance of reflective learning strategies in mathematics education. As Larrivee (2009) and Boaler (2016) noted, reflective practices cultivate metacognitive awareness and foster deeper engagement with mathematical concepts. By aligning textbook design with reflective learning principles, curriculum developers can enhance instructional materials to promote higher-order thinking and conceptual mastery.

## **STUDY LIMITATIONS**

The two mathematics textbooks from PNG and Japan in our study might not fully represent all textbooks used in both countries. The analysis is based on examining the content structure rather than classroom interactions. Future studies should explore how students and teachers engage with textbooks in real learning environments.

Differences in educational approaches and cultural contexts may impact how reflection is encouraged, suggesting the need for broader research across curricula and grades.

## CONCLUSION

This study compared mathematics textbooks from Japan and PNG to explore how they promote reflective learning on the topic of fractions. Reflection is vital in mathematics education, as it enhances students' understanding by encouraging critical thinking and problem-solving skills. We noticed the following important features in each textbook: a blue question mark (?) icon for the Japanese textbook and a purple fast forward (▶▶) icon for the PNG textbook, both near the key question/statement signifying previously learned lessons for the benefit of learners having difficulties in understanding the topic. Conversely, an obvious difference is that a student's picture is used in the Japanese textbook for learners to visualize and relate to, while an object is used in the PNG textbook for learners to visualize and relate to, to explain the mathematical concept of fractions. Lortie-Forgues et al. (2015) showed that using culturally contingent variation in instruction and prior knowledge of learners influences the likelihood of children overcoming difficulties and mastering fractions and decimal arithmetic. Learners develop better understanding by observing scenarios that are familiar, culturally significant, and related to real-life, rather than unfamiliar objects.

The analysis revealed that Japanese textbooks provide opportunities to foster reflection through structured arrangements of questioning methods and discussion. These textbooks guide students in reflecting on the key concepts of a topic and deepen their understanding beyond task completion. By contrast, PNG textbooks could benefit from incorporating these strategies to build understanding of the topic. Students are restricted from realizing the topic's key concept because of unfamiliar objects with which to make comparisons. Thus, encouraging reflection in mathematics helps students grasp complex concepts, such as fractions, strengthening their overall mathematical understanding. When learners reflect, they can develop their understanding and resort to previously learned concepts to help them understand more difficult concepts as they progress to higher grades.

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